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RE-26

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ABSTRACT

The problem of optimum utilization of star tracker data to monitor and correct for the drift of a gyro stabilized orientational reference platform is addressed using minimum variance estimation theory. The system configuration is arranged to make the drift compensation problem independent of base motion isolation errors or the dynamics of the platform servo drive system. A complete analytic solution to the problem is given with star tracker errors modeled as wide band noise and gyro drift rate modeled as a random walk. The time histories of the required system gains and the error variances are calculated and plotted for the case of no prior information regarding gyro drift rate or orientational error, and for the case of no prior information on orientational error but steady state knowledge of gyro drift rate due to previous star tracking. These results should be useful as a guide to the design of stellar-aided inertial platforms and as a reference against which to compare their performance.

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INTRODUCTION

Among the possible means of holding an inertial orientation reference on board an aerospace vehicle, the most useful techniques are gyroscope stabilization and tracking the lines of sight to stars. Each technique has its major strength and weakness - and these characteristics tend to complement each other. The gyro provides wide bandwidth information about angular rotations with respect to inertial space. A gimbaled platform servo-driven to null the outputs of such gyros can maintain an inertial reference orientation to good accuracy even in the presence of high frequency rotations of the vehicle which carries the platform. In long-term operation, however, the gyro suffers from drift which, although it may be a very small angular rate, becomes significant when integrated over a long time. A star tracker, on the other hand, senses the line of sight to a star and indicates an inertial reference direction with a quality which does not deteriorate with time. The bandwidth of a star tracker stabilized platform is, however, more restricted than that of a gyro stabilized platform - and the attempt to increase the bandwidth of such a system is inevitably accompanied by an increased transfer of detector and other noise.

The complementary characteristics of these two types of stabilization system lead one naturally to consider a combination system: a gyro stabilized platform which provides wide band isolation from vehicle rotations, and a star tracker which provides a long-term orientational reference. Such a system is shown in functional diagram form in Fig. 1. The star tracker provides a noisy measure of the platform error angle. From this indication is derived a signal which produces a torque on the platform-mounted gyro. The purpose of this commanded torque is to balance the gyro drift as well as possible, and thus to maintain the stabilization error small over the period of system operation.

The problem to which this paper is addressed is the design of the Drift Compensation Function as shown in Fig. 1. What is the best functional relationship to use in deriving the gyro torquing signal from the star tracker indication of error angle? And what are the best numerical values of the parameters in this function? Statistical filtering theory provides the answers to these questions if "best" is interpreted as minimum RMS errors. Stellar-aided inertial systems have been discussed in previous papers. Blumhagen[1] selected the Drift Compensation Function on an intuitive basis and calculated the resulting performance of the complete navigator. Dusek[3] assumed that the use of star tracking rendered gyro drift unimportant and employed Kalman filtering to estimate accelerometer bias. The contribution of this paper is an analytic optimum solution to the problem of stellar monitoring of a gyro stabilized platform with a fairly realistic mathematical model of the physical situation. The analytic solution permits one to design the Drift Compensation Function immediately for star trackers and gyroscopes of any quality and for any degree of prior information about the stabilization error and gyro drift rate.

THE MODEL OF THE PROBLEM

Maintaining an orientational reference is a three-axis problem, but with the understanding that all three axes are well stabilized, the problem can be treated as a set of three essentially uncoupled stabilization problems. The single axis problem is considered here. The gyro stabilized platform involves the dynamics of the servo loop which drives the gimbal-mounted platform so as to null the output of the gyro. The platform orientation may be in error due both to the failure of this servo to hold the gyro output angle at null and to the fact that an error might still exist even if the platform were rotated to null the gyro output. The former is the base motion isolation error which is the responsibility of the gimbal drive servo system. This system is wide band, and the base motion isolation function cannot effectively be improved by use of star tracker data. The second source of error, that which would exist even if the gimbal drive servo brought the gyro output angle to zero, will be called the stabilization reference error. It is due to initial condition errors and accumulated gyro drift. This is the error which the star tracker is used to control. The stabilization reference error is written

$$\theta_e = \theta_s - \theta_p + \theta_g \tag{1}$$

The notation is defined in Fig. 1. This is the platform error, $\theta_s - \theta_p$, with the gyro output angle, θ_g, added. Addition of the known part of the stabilization error, θ_g, eliminates the dynamics of base motion isolation from the problem.

The dynamics of the gyro are trivial in this problem which is concerned with long-term errors, so the rate of change of gyro angle is that of the platform, plus the precession due to the torquing signal, plus the drift rate, D.

$$\dot{\theta}_g = \dot{\theta}_p + T + D \tag{2}$$

Noting that the star line angle is fixed, the rate of change of stabilization reference error is found from (1) and (2) to be

$$\dot{\theta}_e = T + D \tag{3}$$

Gyro drift is usually modeled as having a random component plus components which depend both linearly and quadratically on the acceleration of the instrument. In this problem, we model only the random component, which implies either space flight operation in free fall, or adequate calibration of the specific force dependent components. The random drift rate might be modeled in different ways, but the most important characteristic is a random walk[2]. Use of just a random walk to model the gyro drift rate permits an analytic solution and preserves the dominant nature of the problem.

$$\dot{D} = n(t)$$

$$n(t) = \text{an unbiased white noise} \tag{4}$$

$$\langle n(t_1)\, n(t_2) \rangle = N \ \delta(t_2 - t_1)$$

The coefficient, N, in the autocorrelation function for the white noise is also the (constant) power spectral density for the noise. Its numerical value is best determined from the resulting mean squared value of gyro drift.

$$\langle D\ (t)^2 \rangle = Nt \tag{5}$$

Thus, for example, if the RMS drift rate after one hour of operation following calibration were supposed to be 10^{-2} deg/hr, the appropriate value of N would be 10^{-4} $deg^2\ hr^{-3}$.

The star tracker indicates the angular error between the star line and the platform reference line. But this indication is corrupted with noise which is considered additive. No initial filtering of the noise is presumed done as any desirable filtering will appear in the Drift Compensation Function to be determined. Hence, the noise is surely wide band compared with the bandwidth of the over-all system, and can reasonably be approximated as a white noise.

$$\begin{aligned} \theta_{ind} &= \theta_s - \theta_p + r(t) \\ r(t) &= \text{unbiased white noise} \\ \langle r(t_1)\, r(t_2) \rangle &= R\,\delta(t_2 - t_1) \end{aligned} \qquad (6)$$

The actual star tracker noise is a wide band noise rather than a truly white noise, and R is chosen to be the power density of the actual noise in the low frequency region.

The system diagram corresponding to this model is shown in Fig. 2.

THE FORM OF THE COMPENSATOR

The error compensation is derived by first determining the optimum estimate of the stabilization reference error from the star tracker indication, and then developing a gyro torquing signal which will drive the estimated error to zero. The minimum variance estimator as derived by Kalman and Bucy[5] has the following general form:

System dynamics: $$\dot{\underline{x}} = F\underline{x} + \begin{bmatrix} T \\ 0 \end{bmatrix} + Gn \quad (7)$$

Measurement: $$m = H\underline{x} + r \quad (8)$$

Estimator: $$\frac{d}{dt}\hat{\underline{x}} = F\hat{\underline{x}} + \begin{bmatrix} T \\ 0 \end{bmatrix} + EH^TR^{-1}(m - H\hat{\underline{x}}) \quad (9)$$

Covariance equation: $$\dot{E} = FE + EF^T + GNG^T - EH^TR^{-1}HE \quad (10)$$

All errors are considered to have zero mean, or to have mean values calibrated out. The gyro torquing signal, T, appears both in the system dynamics and in the estimator. The gyro is presumed to respond perfectly to this command; thus it plays no role in the estimation process. The present problem is put into this general form by means of the following definitions:

$$\underline{x} = \begin{bmatrix} \theta_e \\ D \end{bmatrix} \qquad m = \theta_{e\,ind}$$

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad H = [1 \quad 0] \quad (11)$$

The separate elements of the covariance matrix for the estimation errors are also identified.

$$e_{11} = \langle (\hat{\theta}_e - \theta_e)^2 \rangle$$

$$e_{12} = e_{21} = \langle (\hat{\theta}_e - \theta_e)(\hat{D} - D) \rangle \qquad E = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \qquad (12)$$

$$e_{22} = \langle (\hat{D} - D)^2 \rangle$$

Use of the definitions (11) and (12) in (9) gives

$$\frac{d}{dt}\hat{\theta}_e = \hat{D} + T + \frac{1}{R} e_{11} (\theta_{e\,ind} - \hat{\theta}_e) \qquad (13)$$

$$\frac{d}{dt}\hat{D} = \frac{1}{R} e_{21} (\theta_{e\,ind} - \hat{\theta}_e) \qquad (14)$$

These are the differential equations for the optimum estimates of the stabilization reference error and gyro drift rate respectively.

Rather than just estimate the stabilization reference error, we wish to drive the estimated error to zero. The gyro torquing signal is used for this purpose. Generate T so that $\hat{\theta}_e = 0$ at all times; hence $\frac{d}{dt}\hat{\theta}_e = 0$ also. Equations (13) and (14) then become

$$T = -\hat{D} - \frac{e_{11}\,\theta_{e\ ind}}{R} \tag{15}$$

$$\frac{d}{dt}\hat{D} = \frac{e_{21}\,\theta_{e\ ind}}{R} \tag{16}$$

These equations define the desired relation between the star tracker indication of the stabilization reference error and the gyro torquing signal. The block diagram of this Drift Compensation Function is shown in Fig. 3. The optimum compensator is seen to be a parallel connection of a proportional and an integral path. The proportional gain is the ratio of the mean squared error in the estimate of the error angle to the power density of the measurement noise. The better the quality of the current estimate of θ_e, the less proportional gain is used to feed the star tracker measurement forward. The integral path results from the estimate of the gyro drift rate. The gain from the angle measurement to the drift rate estimate is proportional to the correlation between the errors in estimates of angle and drift rate. Increased star tracker noise relative to the estimation errors reduces both gains. The integrator in the compensator provides the infinite gain at zero frequency required to compensate the infinite power density of gyro drift at zero frequency. (Note that the power density spectrum of the drift rate is $^{N}/\omega^{2}$.)

SOLUTION FOR THE GAINS

The compensator is now defined in terms of the elements of the error covariance matrix. The differential equation for this matrix, (10), has a known solution form[4], but the analytic manipulations are simpler if an alternate approach is taken. The problem can be reduced to the solution of an algebraic matrix quadratic equation and a linear matrix differential equation[6]. First, define S to be a solution of the algebraic equation

$$FS + SF^{T} + GNG^{T} - SH^{T}R^{-1}HS = 0 \tag{17}$$

Then express E as

$$E = S + \delta E \tag{18}$$

The differential equation for δE is found from (10) to be

$$\delta\dot{E} = \tilde{F}\delta E + \delta E\,\tilde{F}^{T} - \delta E\,H^{T}R^{-1}H\,\delta E \tag{19}$$

where

$$\tilde{F} = F - SH^{T}R^{-1}H \tag{20}$$

Finally, define

$$P = \delta E^{-1} \tag{21}$$

Then the differential equation for P is

$$\dot{P} = - P\,\delta\dot{E}\,P$$
$$= - P\tilde{F} - \tilde{F}^T P + H^T R^{-1} H \qquad (22)$$

which is linear and has a known solution form.

A particular solution, P_∞ , to (22) is found by setting $\dot{P} = 0$. This yields

$$P_\infty \tilde{F} + \tilde{F}^T P_\infty = H^T R^{-1} H \qquad (23)$$

The complete solution to (22) can then be written

$$P(t) = \Phi(t) \left[P(0) - P_\infty \right] \Phi(t)^T + P_\infty \qquad (24)$$

where $\Phi(t)$ is the state transition matrix defined by

$$\dot{\Phi} = - \tilde{F}^T \Phi \qquad \Phi(0) = I \qquad (25)$$

If the eigenvalues of $-\tilde{F}$ lie in the left half plane, P_∞ corresponds to the steady state value of $P(t)$.

In general, equation (17) has $(2n)!/(n!)^2$ distinct solutions[7] where n is the dimension of the state vector. Some of these solutions may be nonsymmetric or contain complex elements. Any solution of (17) may be employed to obtain equation (22) and hence the time history of E(t). Since the solution to equation (10) for a given initial condition is unique, the same time history will be obtained regardless of which solution of (17) is employed. If the state is observable by the measurements and controllable by the process noise, equation (17) has unique positive and negative definite solutions (Ref. 6 for positive definite solution). For these positive and negative definite solutions, the matrices $\widetilde{F}$ and $-\widetilde{F}$ respectively have eigenvalues in the left half plane (correspond to stable systems). From the standpoint of algebraic manipulation, the positive and negative definite solutions seem to be the easiest to work with. In the present problem, the negative definite solution is employed in order to simplify the formulation of the initial condition for (22) in case II. If the positive definite solution had been employed, the initial value, P(0), would have contained an infinite element and a limiting process would have to have been carried along throughout the algebra.

Having the solution for P(t), E(t) is written

$$E(t) = S + P(t)^{-1} \tag{26}$$

For the problem at hand, the required matrices are defined by (11). R and N, in this case, are the scalar power densities for the star tracker noise and the gyro drift random walk generator respectively. Of the two real symmetrical solutions to (17) in this case, one is the positive definite steady state value of the error covariance matrix.

$$E_\infty = \begin{bmatrix} 2^{\frac{1}{2}} N^{\frac{1}{4}} R^{\frac{3}{4}} & (NR)^{\frac{1}{2}} \\ (NR)^{\frac{1}{2}} & 2^{\frac{1}{2}} N^{\frac{3}{4}} R^{\frac{1}{4}} \end{bmatrix} \tag{27}$$

Take S to be the negative definite solution.

$$S = \begin{bmatrix} -2^{1/2} N^{1/4} R^{3/4} & (NR)^{1/2} \\ (NR)^{1/2} & -2^{1/2} N^{3/4} R^{1/4} \end{bmatrix} \tag{28}$$

Then $\tilde{F}$ given by (20) is

$$\tilde{F} = \begin{bmatrix} 2^{1/2} N^{1/4} R^{-1/4} & 1 \\ -N^{1/2} R^{-1/2} & 0 \end{bmatrix} \tag{29}$$

and the solution to (23) is

$$P_\infty = \begin{bmatrix} 2^{-3/2} N^{-1/4} R^{-3/4} & 0 \\ 0 & 2^{-3/2} N^{-3/4} R^{-1/4} \end{bmatrix} \tag{30}$$

It may be verified that E_∞ is properly given by the sum of S and the inverse of P_∞ . Integration of (25) yields

$$\Phi(t) = \begin{bmatrix} \cos at - \sin at & 2a \sin at \\ -\frac{1}{a} \sin at & \sin at + \cos at \end{bmatrix} e^{-at} \tag{31}$$

where

$$a = 2^{-1/2} N^{1/4} R^{-1/4} \tag{32}$$

The solution will now be written out for two cases.

Case I: No initial knowledge of error angle or drift rate.

One case of practical importance is that in which there is no prior information on the stabilization error angle or gyro drift rate. Of course, it is necessary that the star be acquired within the field of view of the star tracker, but that field is considered wide relative to the desired stabilization reference accuracy. Hence, the initial estimation error variances are considered essentially infinite, or P(0) is the zero matrix. This initial condition would typically apply when the system is first employed following a long delay since a calibration. This would be the case if the platform were to be used in the vicinity of a planet after having been inactive during the journey from Earth. With P(0) = 0, equation (24) becomes

$$P(t) = \frac{1}{4aR}\begin{bmatrix} 1-(1+2s^2-2sc)e^{-2at} & -\frac{2s^2}{a}e^{-2at} \\ -\frac{2s^2}{a}e^{-2at} & \frac{1}{2a^2}\left[1-(1+2s^2+2sc)\right]e^{-2at} \end{bmatrix} \tag{33}$$

where

$$\begin{aligned} s &= \sin at \\ c &= \cos at \end{aligned} \tag{34}$$

The solution for E(t), from equation (26), is

$$e_{11} = 2aR\,\frac{1-4sce^{-2at}-e^{-4at}}{(1-e^{-2at})^2-4s^2e^{-2at}} \tag{35}$$

$$e_{21} = 2a^2R\,\frac{(1-e^{-2at})^2+4s^2e^{-2at}}{(1-e^{-2at})^2-4s^2e^{-2at}} \tag{36}$$

$$e_{22} = 4a^3 R \frac{1 + 4sc\, e^{-2at} - e^{-4at}}{(1 - e^{-2at})^2 - 4s^2 e^{-2at}} \tag{37}$$

Alternate forms for these expressions are

$$e_{11} = aR \frac{\sinh 2at - \sin 2at}{\sinh^2 at - \sin^2 at} \tag{38}$$

$$e_{21} = 2a^2 R \frac{\sinh^2 at + \sin^2 at}{\sinh^2 at - \sin^2 at} \tag{39}$$

$$e_{22} = 2a^3 R \frac{\sinh 2at + \sin 2at}{\sinh^2 at - \sin^2 at} \tag{40}$$

These covariance matrix elements are plotted in Fig. 4. The variance of the stabilization reference error, e_{11}, and of knowledge of gyro drift rate, e_{22}, both start at infinitely large values according to the assumption of no prior information. Their covariance, $e_{21} = e_{12}$, does likewise. All elements reduce monotonically toward their steady state values, coming within 10 percent of their final values in the neighborhood of at = 2. The gains in the drift compensation function are proportional to e_{11} and e_{21}. These gains would, of course, be restricted to finite values. This limitation should incur little loss of performance since the ideal proportional gain, is within a factor of 100 times its steady state value by at = 0.02. The integral gain reaches this same factor by at = 0.18.

As an example of the magnitude of the inverse time constant, a, take N to be 10^{-4} deg^2 hr^{-3} which gives a gyro drift rate random walk having an RMS value of 10^{-2} deg/hr after 1 hour. Also, suppose the star tracker noise is a wide band process yielding an RMS value of 1 milliradian through a 1 cps bandwidth. R is then $10^{-6}rad^2sec$. For these values,

$$
\begin{aligned}
a &= 2^{-1/2} N^{1/4} R^{-1/4} \\
&= \left(\frac{10^{-4}\cdot 3600}{4\cdot 10^{-6}\,(57.3)^2}\right)^{1/4} \\
&= 2.29\ hr^{-1}
\end{aligned}
$$

With these illustrative figures, the transient period for the covariance matrix elements and for the Drift Compensation Function gains is about one hour in duration. This transient period increases if the performance of the star tracker is degraded relative to that of the gyro.

Case II: No initial knowledge of error angle but gyro drift estimation at steady state.

This case occurs whenever system operation has reached its steady state, and then a reorientation of the platform is called for. If the reorientation cannot be accomplished with precision, we may take the initial variance of the error angle to be essentially infinite while the initial variance of the drift rate estimate remains at its steady state value.

$$e_{11}(0) = \infty \qquad\qquad e_{22}(0) = 4a^{3}R \tag{41}$$

Now

$$P(0) = \left[E(0) - S\right]^{-1} \tag{42}$$

Using S from equation (28) and the initial conditions (41), the initial value of P is independent of $e_{21}(0)$ and is found to be

$$P(0) = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{8a^3R} \end{bmatrix} \tag{43}$$

Equation (24) then gives

$$P(t) = \frac{1}{4aR}\begin{bmatrix} 1-(1-2sc)e^{-2at} & \frac{1}{a}s(c-s)e^{-2at} \\ \frac{1}{a}s(c-s)e^{-2at} & \frac{1}{2a^2}\left(1-2s^2e^{-2at}\right) \end{bmatrix} \tag{44}$$

using the notation (34). The covariance matrix elements are then expressed by (26) as

$$e_{11} = 2aR\,\frac{1+(1-2sc-2s^2)e^{-2at}}{1-(1-2sc+2s^2)\,e^{-2at}} \tag{45}$$

$$e_{21} = 2a^2 R \frac{1-(1+2sc-2s^2)e^{-2at}}{1-(1-2sc+2s^2)e^{-2at}} \tag{46}$$

$$e_{22} = 4a^3 R \frac{1-(1-2sc-2s^2)e^{-2at}}{1-(1-2sc+2s^2)e^{-2at}} \tag{47}$$

or alternatively as

$$e_{11} = 2aR \frac{1+(\cos 2at - \sin 2at)e^{-2at}}{1-(2-\cos 2at - \sin 2at)e^{-2at}} \tag{48}$$

$$e_{21} = 2a^2 R \frac{1-(\cos 2at + \sin 2at)e^{-2at}}{1-(2-\cos 2at - \sin 2at)e^{-2at}} \tag{49}$$

$$e_{22} = 4a^3 R \frac{1-(\cos 2at - \sin 2at)e^{-2at}}{1-(2-\cos 2at - \sin 2at)e^{-2at}} \tag{50}$$

These covariance matrix elements are plotted in Fig. 5. The variance of the stabilization reference error, e_{11}, starts at an infinite initial value and the variance of the gyro drift rate estimate, e_{22}, starts at its steady state value according to the prescribed initial conditions. The error angle variance in this case converges within 10 percent of its steady state value by about at = 0.5, 4 times faster than in Case I with no prior information about gyro drift. In fact, the initial transient in e_{11} in this case, for $at < 0.5$, is very closely approximated by the e_{11} transient in Case I with the time scale shortened by a factor of 4.

The value of prior gyro calibration is clearly evident. The other covariance matrix elements come within 10 percent of their steady state values at about at = 2 . Note that the gain from angle measurement to drift rate estimate, which is proportional to e_{21}, starts from a zero initial value. This is due to the poor initial knowledge of error angle compared with the finite variance of the initial drift rate estimate.

APPROXIMATE EXPRESSIONS FOR THE GAINS

The proportional gain in the Drift Compensation Function, as indicated in Fig. 3, is e_{11}/R. The integral gain is e_{21}/R. Exact analytic expressions for these gains are thus given by the covariance matrix expressions derived above. As an aid to practical implementation of such a system, approximate expressions for these gains which are of simpler form are suggested here.

Proportional gain $\approx 2a\left(1 + \frac{2}{at} e^{-0.7at}\right)$ (Case I)

$\approx 2a\left(1 + \frac{1}{2at} e^{-2.8at}\right)$ (Case II)

Integral gain $\approx 2a^2\left(1 + \frac{3}{(at)^2} e^{-0.5at}\right)$ (Case I)

$\approx 2a^2\left(\frac{1 + e^{-at}}{1 + \frac{2}{at} e^{-2at}}\right)$ (Case II)

Over most of the time interval ($0.01 \leq at < \infty$) these expressions are in error by less than a few percent. The maximum error in this interval is 14.3 percent.

CONCLUSIONS

The design of a star tracker aided inertial orientational reference system has been determined which minimizes at every time the mean squared error angle. The error referred to is the platform orientational error which would exist if the stabilization servo system drove the gyro output angle to zero. The compensation of gyro drift is thus rendered independent of platform dynamics and imperfect base motion isolation. The statement of the design problem includes a reasonable statistical characterization of the gyro drift rate and the star tracker noise. A complete analytic solution to this problem has been found which provides analytic expressions for all required system parameters as general functions of the relative quality of the gyro and star tracker. These results should be useful as a guide to the design of practical systems of this type and as a reference against which to compare their performance.

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FIGURE CAPTIONS

1. Functional Diagram of Orientation Stabilization System
2. Block Diagram of the System Model
3. The Optimum Drift Compensation Function
4. Covariance Matrix Elements

   Case I: No initial information
5. Covariance Matrix Elements

   Case II: No initial stabilization error information,
   gyro drift estimation at steady state

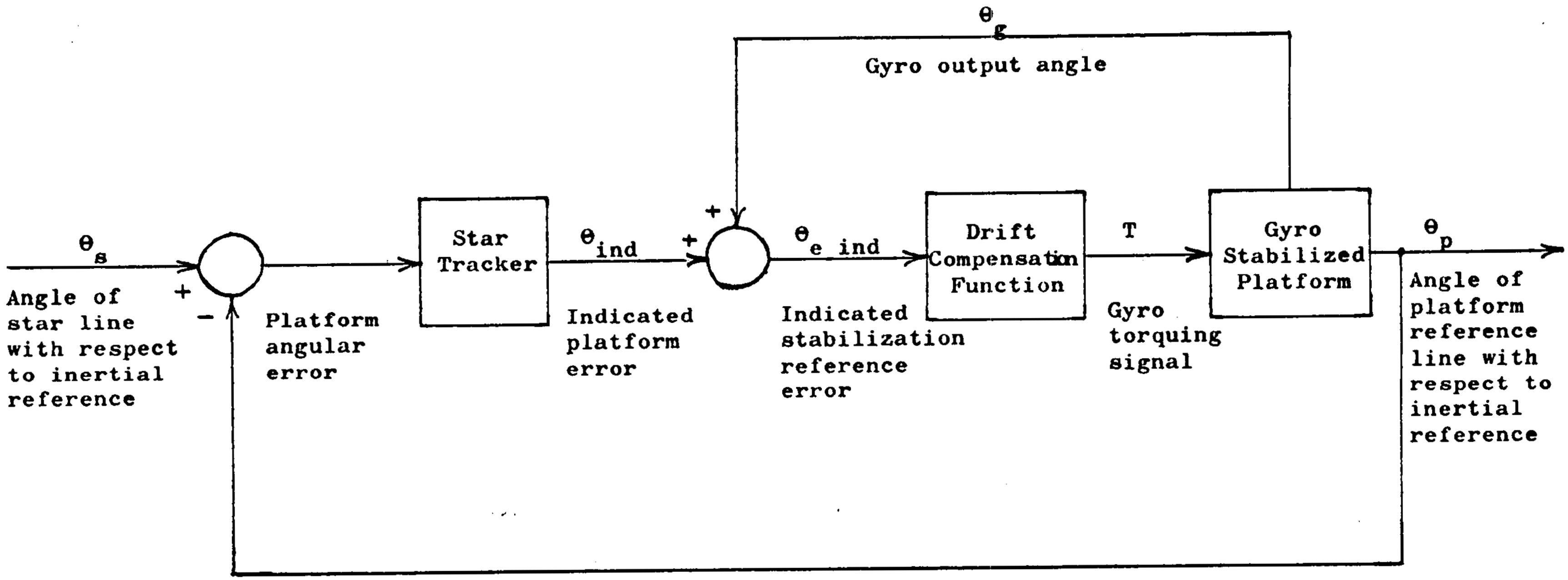

Figure 1 Functional Diagram of Orientation Stabilization System

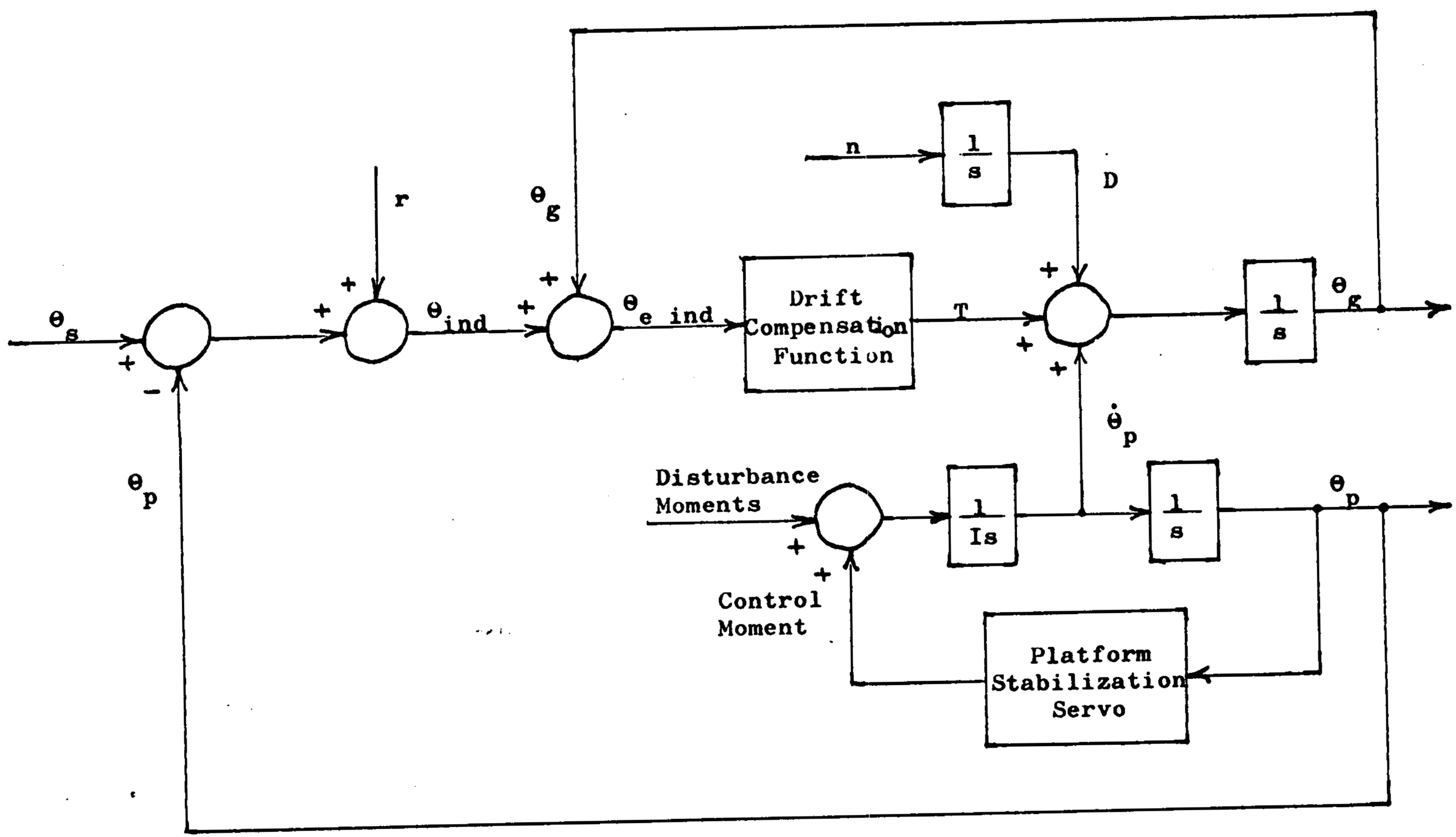

Figure 2 Block Diagram of the System Model

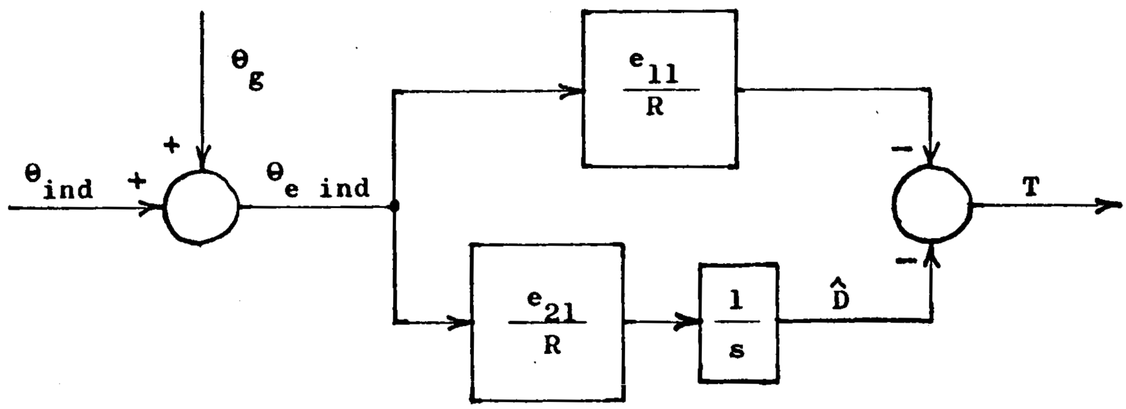

Figure 3 The Optimum Drift Compensation Function

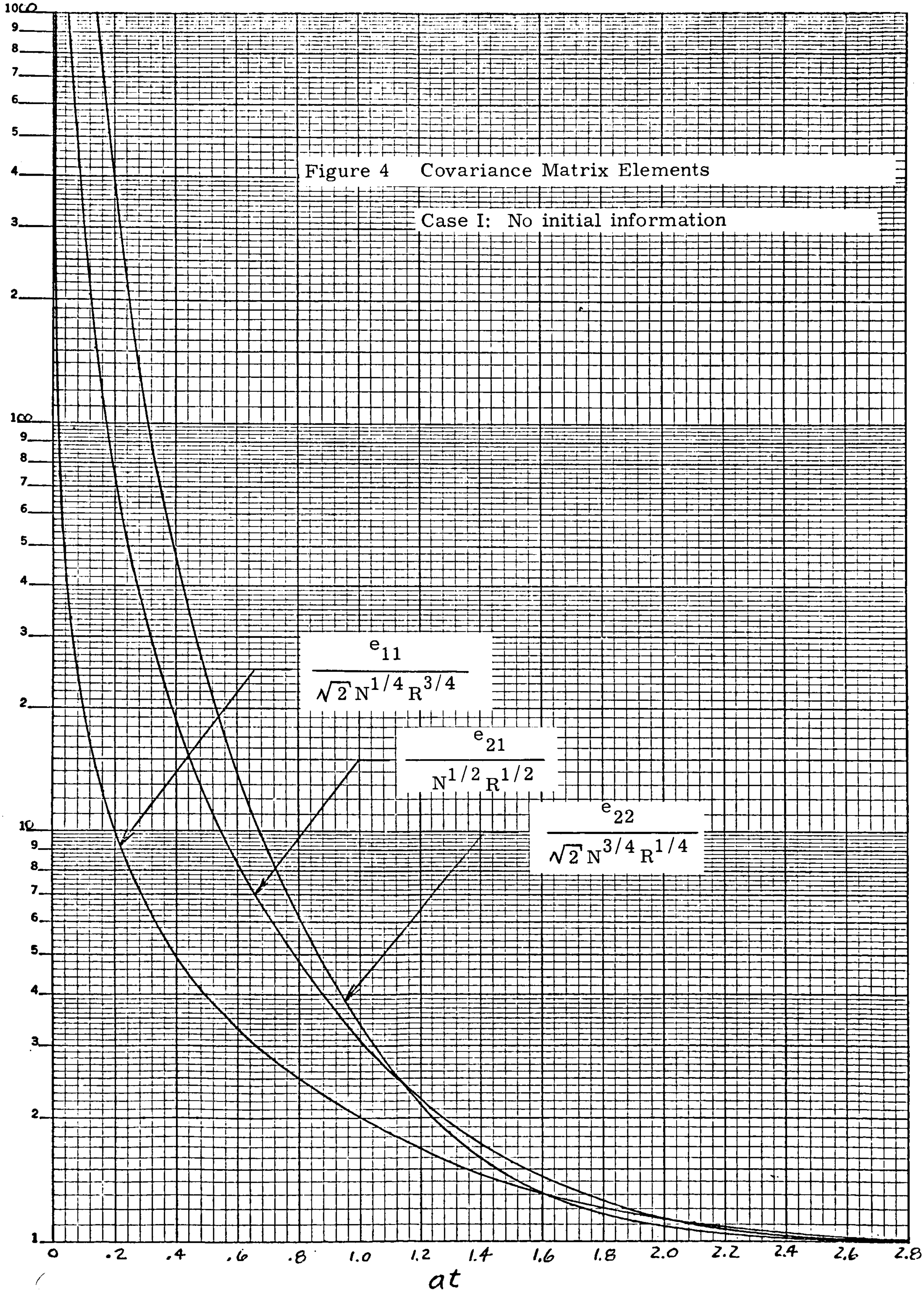
Figure 4 Covariance Matrix Elements
Case I: No initial information
$\frac{e_{11}}{\sqrt{2}\,N^{1/4}R^{3/4}}$
$\frac{e_{21}}{N^{1/2}R^{1/2}}$
$\frac{e_{22}}{\sqrt{2}\,N^{3/4}R^{1/4}}$
1000
100
10
1
0 .2 .4 .6 .8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8
at

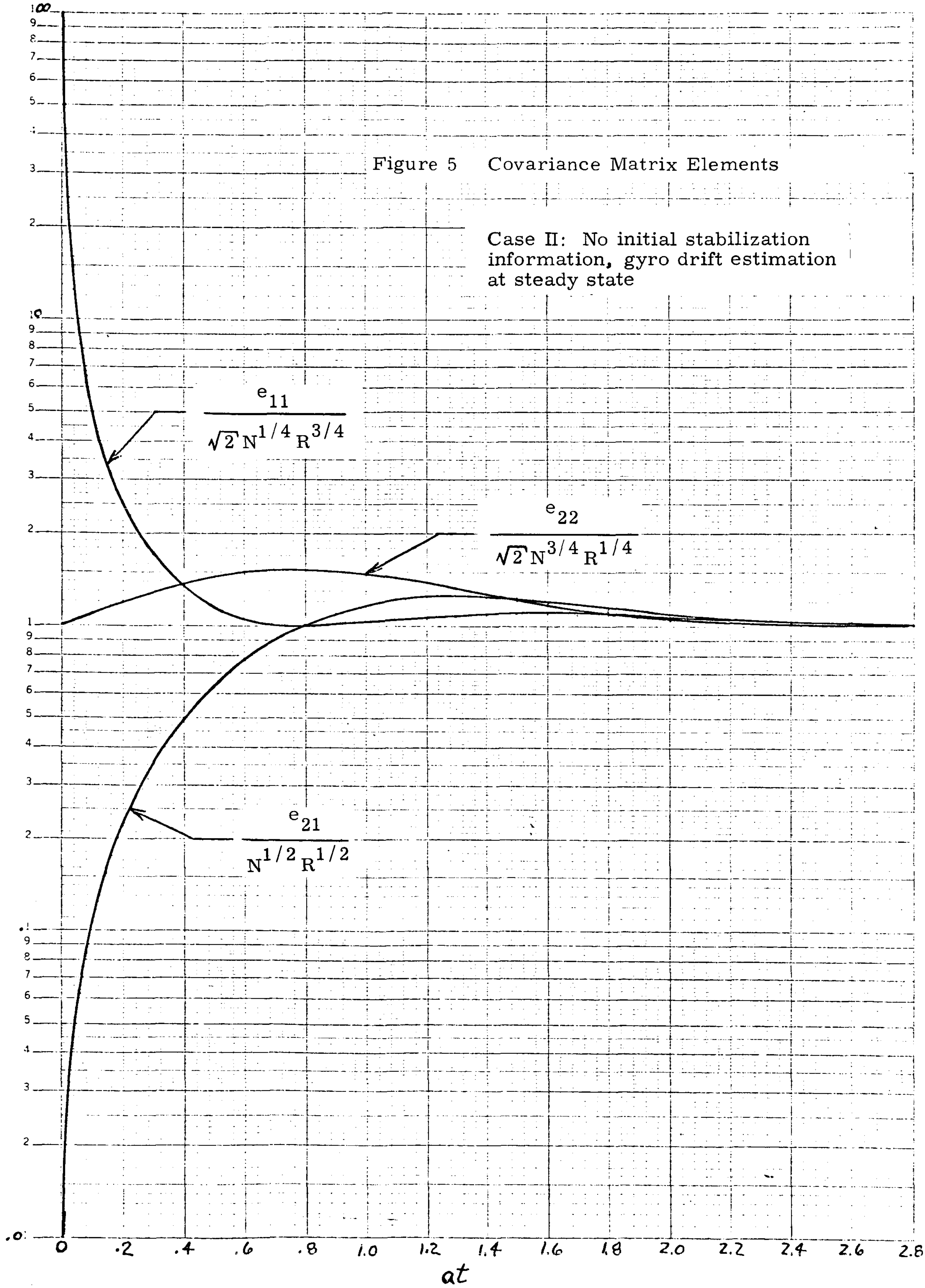

Figure 5 Covariance Matrix Elements